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A MARKOVIAN APPROACH TO MISSILE SYSTEMS TEST SEQUENCING.(U)

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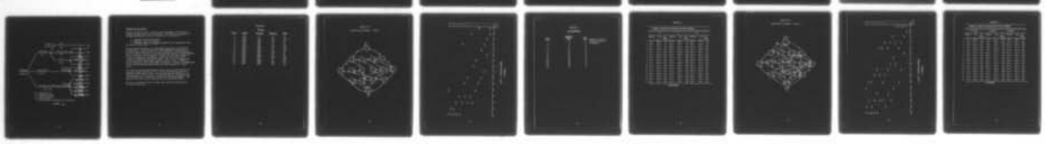

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A MARKOVIAN APPROACH TO MISSILE SYSTEMS TEST SEQUENCING

Colonel Allen F. Grum
Department of Engineering
U.S. Military Academy
West Point, NY

Captain Terry A. Bresnick
Department of Engineering
U.S. Military Academy
West Point, NY

Captain Bruce F. Rogers
U.S. Army Operational Test and Evaluation Agency
Falls Church, VA

ABSTRACT

Given a new missile system, a set of required capabilities, and a limited set of testing resources, we desire to verify mission accomplishment or non-accomplishment as efficiently as possible. There are various test environments that can be used ranging from easy tests that yield small amounts of information, to difficult tests that are more prône to failure but can provide more extensive information. This paper uses a Markovian model to investigate optimal test sequencing with a "highest expected reward" decision criterion.

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Given a new missile system, a set of required capabilities, and a limited set of testing resources, ~~we~~ one desires to verify mission accomplishment or non-accomplishment as efficiently as possible. There are various test environments that can be used ranging from easy tests that yield small amounts of information, to difficult tests that are more prone to failure but can provide more extensive information. This paper uses a Markovian model to investigate optimal test sequencing with a "highest expected reward" decision criterion.

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Overview

At times, the designer of a test sequence may be faced with a dilemma. Constraints on equipment, time, money, or other resources may force an abbreviated testing program. Does the test designer then select as a start a relatively "easy" test with high probability of success but a low payoff, or does he select a "tough" test with a higher payoff but a lower probability of success? (Success is defined as meeting the objectives of the test. This would vary from item to item. However, typical criteria might include 60% of the shots but within a 2 inch circle at a range of 200 meters; the truck will climb a 30% grade when loaded with 10,000 pounds; the mobility of the tractor will equal that of the XM-1; etc.)

We first develop a theory to solve this decision problem. The theory rests heavily on results from Markovian analysis. This theory is then applied to a practical problem, the testing of the ZAP missile system.

Purpose

The purpose of this report is to describe an approach to the test designer's decision problem when the test environment can be modelled as a Markov process and when an "expected reward" decision criterion can be used. The thrust of the report is tutorial.

The Model

We assume a set of N possible discrete test states, 1,2,3,..., N-1,N, with the higher number connoting a more demanding test with a greater reward but a lower probability of completing the test successfully.

We also hypothesize that the progression from test state to test state satisfies the Markovian assumption. We explicitly state the assumption by letting

$$s(n) = j \qquad 0 \leq n \leq \infty, \qquad 1 \leq j \leq N$$

where $s(n)$ represents the test state at the n^{th} transition from test to test, and j is the identifier of the state.

The Markovian Assumption requires that the probability of transitioning from state to state be solely dependent on the state presently occupied. Mathematically, we represent this as

$$\Pr\left\{s(n+1)=j \mid s(n)=i\right\} = \Pr\left\{s(n+1)=j \mid s(n)=i, s(n-1)=k, \ldots, s(0)=m\right\} = p_{ij} \qquad (1)$$

The use of the transition probabilities is completely flexible and enables the analyst to capture any test sequencing decision rules. As examples, suppose that the present state is state i. "Complete" success might call for state k to be the next state; "Partial" success might call for state j to be the next state. Similiarly, "complete" failure might call for a return to state 1 and "partial" failure to state 2. Implied in this description is

$$1 < 2 < i < j < k$$

These rules are described by the analyst a priori assigning values to

p_{i1} - Complete failure

p_{i2} - Partial failure

p_{ij} - Partial success

p_{ik} - Complete success

We may particularize this concept by describing a four state test program as shown in Figure 1. This figure depicts various probabilities of moving from test state to test state. In particular we note failure of test or state 2 moves the test back to test 1; failure of 3 may lead to a repeat of 3; and passing state 2 may lead to either 3 or 4.

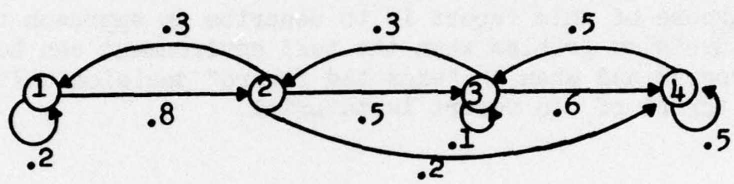


Figure 1

The transition matrix, $[P]$, is nothing more than a tabulation of the various p_{ij}'s as defined by Equation (1).

For our particular situation,

$[P]$ = (2)

From Test State i	To Test State j: j = 1	2	3	4
i = 1	0.2	0.8	0	0
2	0.3	0	0.5	0.2
3	0	0.3	0.1	0.6
4	0	0	0.5	0.5

We have previously noted that higher rewards accrue for completion of higher numbered tests. Therefore, we arbitrarily assign the following rewards

		To Test State j				
		j =	1	2	3	4
From Test State i	i =					
	1		3	5	-	-
	2		0	-	7	9
	3		-	0	2	12
	4		-	-	0	15

(3)

We are now in a position to discuss the solution methodology.

The Methodology

The Markov model has existed since 1907. However, solution difficulties have limited its application, and the model, to a certain extent, has remained a mathematical curio. The advent of the computer has now made practical the matrix manipulations necessary for the solutions. For our particular case we define a variable

$$\nu_{ij}(n)$$

as the number of times test state j is occupied through n transitions given that the process started in the state i.

We need the expected value $\overline{\nu}_{ij}(n)$ and first require

$$\phi_{ij}(n) = \Pr\left\{s(n)=j \mid s(0)=i\right\} \tag{4}$$

or the probability that after transition n, the item will be in state j conditioned on starting in state i. By definition, $\phi_{ij}(n)=0$ for $n<0$, $\phi_{ij}(0)=0$ for $i \neq j$, and $\phi_{ij}(0)=1$ for $i=j$.

$$\left[\Phi(n)\right] = \begin{bmatrix} \phi_{11}(n) & \phi_{12}(n)\ldots & \phi_{1N}(n) \\ \vdots & & \\ \phi_{N1}(n) & \phi_{N2}(n)\ldots & \phi_{NN}(n) \end{bmatrix} \tag{5}$$

One of the classic results of Markov theory is that

$$\left[\underline{\Phi}(n)\right] = [P] \cdot [P] \cdots\cdots [P] \tag{6}$$

$$= [P]^n$$

where $[P]^{\circ} = [I]$, the identity matrix.

The calculations of the $\left[\Phi\right]$ matrix is only a means to the end, the end being the calculation of $\overline{\nu}_{ij}(n)$, the expected state occupancies. We calculate these by defining an indicator random variable $X_{ij}(n)$ such that

$$X_{ij}(n) = \begin{bmatrix} 1 & \text{if } s(n)=j \\ 0 & \text{otherwise} \end{bmatrix} \text{conditioned on } s(0) = i \tag{7}$$

Then the expected value of X_{ij} is

$$\overline{X}_{ij}(n) = 1 \cdot \phi_{ij}(n) + 0 \cdot \left[1 - \phi_{ij}(n)\right] \tag{8}$$

$$= \phi_{ij}(n)$$

We also reason that

$$\nu_{ij}(n) = \sum_{m=0}^{n} X_{ij}(m) \tag{9}$$

and

$$\overline{\nu}_{ij}(n) = \sum_{m=0}^{n} \overline{X}_{ij}(m)$$

$$= \sum_{m=0}^{n} \phi_{ij}(m) \tag{10}$$

Letting

$$\left[\overline{V}(n)\right] = \begin{bmatrix} \overline{\nu}_{11}(n) & \overline{\nu}_{12}(n) & \cdots & \overline{\nu}_{1N}(n) \\ \vdots & & & \vdots \\ \overline{\nu}_{N1}(n) & \cdots & \cdots & \overline{\nu}_{NN}(n) \end{bmatrix} \tag{11}$$

$$= \sum_{m=0}^{n} \begin{bmatrix} \phi_{ij}(n) & \phi_{12}(n) & \cdots & \phi_{1N}(n) \\ \vdots & & & \vdots \\ \phi_{N1}(n) & \phi_{N2}(n) & \cdots & \phi_{NN}(n) \end{bmatrix} \tag{12}$$

$$= \sum_{m=0}^{n} [P]^{m} \tag{13}$$

The expected state occupancies for the example problems are shown in Table 1.

Although close to home we still have one or two more steps to complete. The transitions from test state to test state yield different rewards. Consider test state 3 as an example. The [P] matrix (2) shows that 30% of the transitions are to state 2, 10% to a retest at 3, and 60% to test state 4. These probabilities and the associated rewards are shown in Table 2.

State	State Probability	Reward		Expected Reward
2	0.3	0		0
3	0.1	2		.2
4	0.6	12		7.2
			Total	7.4

Table 2

Therefore, the expected reward each time we transition out of state 3 is 7.4 units.

We may generate the entire expected reward matrix, a column matrix, $[\bar{R}]$, by

$$[\bar{R}] = \text{diag } [P] \ [R]^T$$

where $[R]^T$ is the transpose of the $[R]$ matrix and diag implies using only the diagonal elements of the n x n matrix one obtains by multiplying $[P]$ by $[R]^T$.

We must digress to be perfectly precise about "counting". Rewards accrue only on transitions out of a state. The expected state occupancies include a final occupancy with no associated exiting. Therefore, the correct $[\bar{V}(n)]$ is one less than the number of tests to be run, i.e., if the analyst is looking for the optimal starting state conditioned on 12 tests he is interested in $[\bar{V}(11)]$.

Multiplication of $[\bar{R}]$ by $[\bar{V}(n)]$ gives the expected value from each state occupied. We label this $[B(n)]$ or the benefit matrix,

$$[B(n)] = [\bar{V}(n)] \ [\bar{R}]$$

We will illustrate these abstract concepts with a specific example.

TABLE I

Expected Occupancies of State:

	Start in State	1	2	3	4
n = Ø	1	1.ØØØØ	Ø.	Ø.	Ø.
	2	Ø.	1.ØØØØ	Ø.	Ø.
	3	Ø.	Ø.	1.ØØØØ	Ø.
	4	Ø.	Ø.	Ø.	1.ØØØØ
n = 1	1	1.2ØØØ	Ø.8ØØØ	Ø.	Ø.
	2	Ø.3ØØØ	1.ØØØØ	Ø.5ØØØ	Ø.2ØØØ
	3	Ø.ØØØØ	Ø.3ØØØ	1.1ØØØ	Ø.6ØØØ
	4	Ø.ØØØØ	Ø.ØØØØ	Ø.5ØØØ	1.5ØØØ
n = 2	1	1.48ØØ	Ø.96ØØ	Ø.4ØØØ	Ø.16ØØ
	2	Ø.36ØØ	1.39ØØ	Ø.65ØØ	Ø.6ØØØ
	3	Ø.Ø9ØØ	Ø.33ØØ	1.56ØØ	1.Ø2ØØ
	4	Ø.ØØØØ	Ø.15ØØ	Ø.8ØØØ	2.Ø5ØØ
n = 3	1	1.58ØØ	1.3ØØØ	Ø.6ØØØ	Ø.51ØØ
	2	Ø.49ØØ	1.48ØØ	1.Ø6ØØ	Ø.97ØØ
	3	Ø.12ØØ	Ø.54ØØ	1.83ØØ	1.51ØØ
	4	Ø.Ø5ØØ	Ø.24ØØ	1.18ØØ	2.54ØØ
n = 4	1	1.71ØØ	1.45ØØ	Ø.97ØØ	Ø.88ØØ
	2	Ø.54ØØ	1.71ØØ	1.33ØØ	1.42ØØ
	3	Ø.19ØØ	Ø.64ØØ	2.21ØØ	1.96ØØ
	4	Ø.Ø8ØØ	Ø.39ØØ	1.51ØØ	3.Ø2ØØ
n = 5	1	1.78ØØ	1.66ØØ	1.26ØØ	1.31ØØ
	2	Ø.62ØØ	1.83ØØ	1.7ØØØ	1.85ØØ
	3	Ø.23ØØ	Ø.81ØØ	2.52ØØ	2.44ØØ
	4	Ø.13ØØ	Ø.52ØØ	1.86ØØ	3.49ØØ
n = 6	1	1.85ØØ	1.8ØØØ	1.61ØØ	1.74ØØ
	2	Ø.67ØØ	2.Ø1ØØ	2.Ø1ØØ	2.31ØØ
	3	Ø.29ØØ	Ø.94ØØ	2.88ØØ	2.89ØØ
	4	Ø.18ØØ	Ø.66ØØ	2.19ØØ	3.96ØØ

An example

A new missile system, known as ZAP, has been developed and is now in the testing stage. We are interested in determining how the missiles will perform and whether they will meet established criteria.

A test state in our procedure may be defined as a set of factors determining the environment under which a missile is fired. Each factor has two levels, an easy condition and a difficult condition. We will first look at a simple model where the outcome of each experiment is only hit or miss. For the missile system under consideration the factors and conditions are as follows:

Factor	Factor #	Condition
Target speed	1	Slow, fast
Target altitude	2	High, low
Maneuver	3	No, yes
IR Countermeasures	4	No, yes

Our 4 factors and 2 conditions establish 2^4, or 16 test states. These are found in appendix 1. We can further structure these states into five "macro" levels. Level 1 has all factors in the easiest condition; level 2 has each state with one factor at the difficult condition; level 3 has two difficult factors; level 4 has three difficult factors, and level 5 has all factors at maximum difficulty.

A transition diagram showing possible paths can be developed for this scenario such as the one shown in Appendix 2. Missiles are fired individually in accordance with the following rules: if a test is successful, follow path A. If unsuccessful, follow path B. Using a combination of historical data on somewhat similar missile systems, and the knowledge of technical experts, probabilities for each transition were assessed. The $[P]$ matrix is shown at Appendix 3.

By again consulting the technical experts, a relative "benefit" scale for passing tests of varying degrees of difficulty was established and is shown at Appendix 4.

Using the results developed earlier, we then calculate the probabilities of reaching any state as a function of the number of missiles tested and the initial test state. For this example the expected value results yielded a decision rule far different than the testing program currently in use. At present, testing begins at the lowest levels and becomes progressively more difficult. Our model suggests starting the testing process in macro-level 4. Within this level, test state 12 proved to be a slightly better choice. This was true regardless of the number of missiles available to be fired.

Thus far, we have still only discussed the benefits of passing tests with no consideration of costs. Fortunately, this can easily be incorporated in the model. Actual costs have been approximated by our technical experts and the following information is provided:

1) a target presentation costs $1000

2) loss of target (physical intercept or accident) is $82,000.

3) a success may be a technical kill (target recoverable) or a physical intercept (target lost)

4) one out of three targets hit is a physical intercept

5) If a target is missed, not fired at, or a technical kill, there is a 95% chance of recovery.

6) refurbishing costs of recovered targets are $7000.

We can portray the expected cost picture as shown in figure 2.

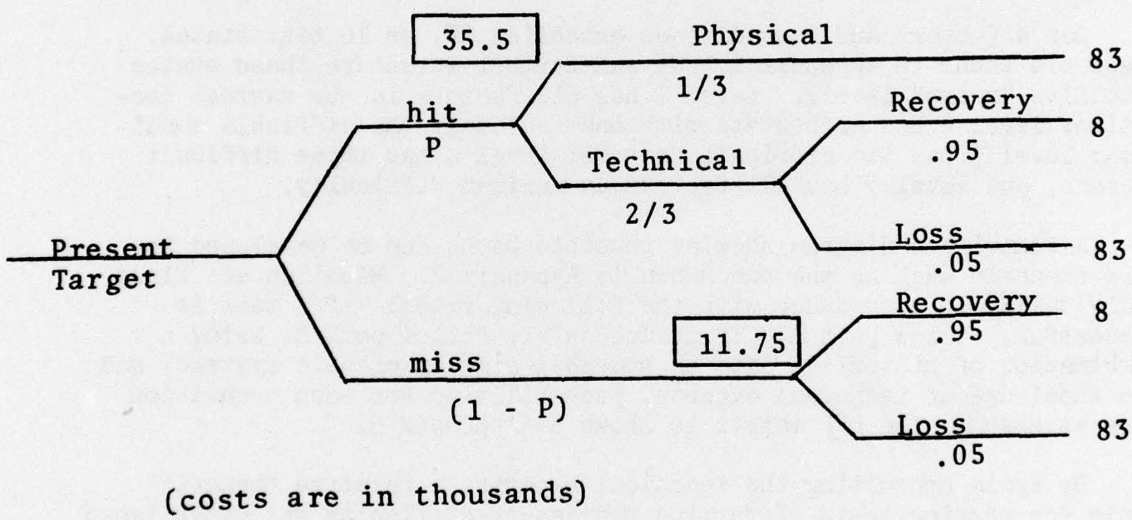


FIGURE 2

Since p is determined by test state, we can derive the expression for expected cost as a function of test state from Figure 2 as:

$$\langle \text{Expected cost} | \text{test state} \rangle = 35.5^k p + 11.75^k (1-p)$$
$$= 11.75^k - 23.75^k p$$

Using a cost oriented decision rule we again evaluated the optimal starting test state. Once again we found that macro-level 4 is the place to start since costs are the cheapest. Within level 4, state 15 had a slight edge. Again, this was independent of number of missiles tested.

To make a decision based upon both cost and benefit, we used a benefit/cost ratio as the value input to the model and again calculated expected utilities. The results were the same macro-level 4 was best on a maximum B/C ratio standard, with state 12 being slightly dominant. The results for specified numbers of missiles are shown for all three decision criteria in Appendix 5.

We next looked at a slightly different testing scheme. Two missiles instead of one are fired in each test environment. The possible outcomes are now 2 misses, 1 hit and 1 miss, or 2 hits. The results of each test determined what the next test state will be. The transition diagram is shown at Appendix 6. If two missiles hit, follow path A, if one hits, follow path B, and if none hit, follow path C. The [P] matrix for this diagram is at Appendix 7. The same benefit scale was used.

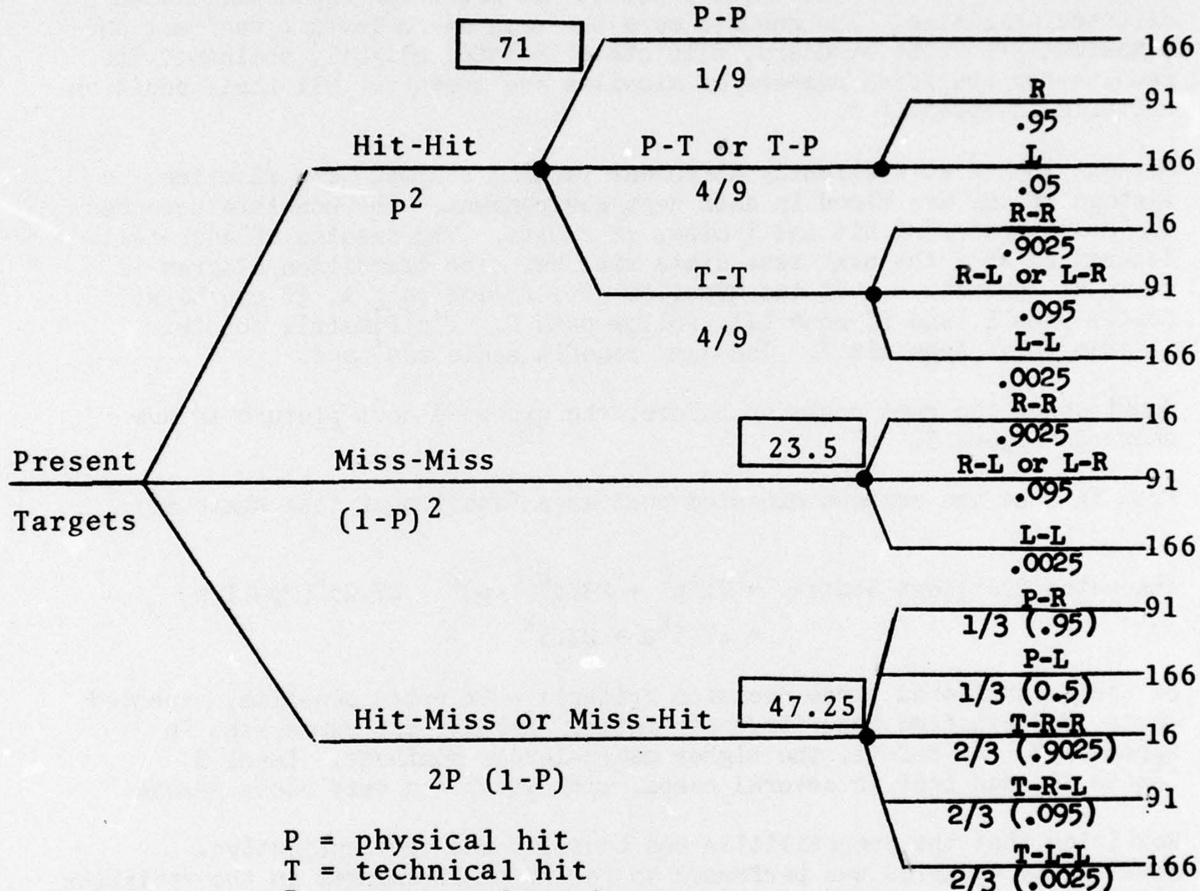
Considering the same costs as before, the expected cost picture is now shown in Figure 3.

From this we can express expected cost as a function of test state as:

$$\langle \text{Expected Cost} \mid \text{Test State} \rangle = 71^k p^2 + 23.5^k (1-p)^2 + 47.25^k (2p)(1-p)$$
$$= 47.5^k p + 23.5^k$$

We again considered three decision criteria - expected benefits, expected costs, and expected benefits/costs ratio. Results are summarized in Appendix 8. As before, the higher macro-levels dominate. Level 5 (state 16) was best in several cases, with level 4 a very close second.

Realizing that the probabilities and benefits are very subjective, sensitivity analysis was performed to see if small changes in the variables critically changed the results. First, probabilities were adjusted to "bias" the model towards lower testing levels. Next, benefits were modified, again with a bias towards lower levels. Finally, both were adjusted simultaneously. In all cases, even though we tried to force the decision rule to change, it was highly insensitive to these modifications. Macro-level 4 was the testing environment that consistently proved to be the best.



P = physical hit
T = technical hit
R = recover target
L = lose target
(costs are in thousands of dollars)

FIGURE 3

Summary and Conclusions

We have developed a model to assist the test designer in his choice of initial starting point in a test sequence. The analysis is dependent on the Markov assumption as well as the ability of the tester to:

1. Identify discrete test states.
2. Enumerate costs and benefits.
3. Determine rules and *a priori* probabilities for progression from test state to test state.

We have applied this model to a hypothetical situation, the testing of the ZAP missle. We have conducted a three dimensional sensitivity in the sense that we have varied (1) costs and benefits, (2) probabilities, and (3) decision criteria. In each case we found, based on the hypothesized decision criteria, that starting at a "high" test state was optimal or, to put our findings in the negative, starting at test state 1, the "normal" starting point was never optimal. This is not to suggest that other test patterns would never logically start in state 1. However, for the ZAP missle, over a wide range of variables, a linear test progression yields expected rewards substantially less than starting in a "high" state.

The model and the method of analysis we have used is quite flexible. We can easily expand the number of test states and change the rules for progression from state to state. We can calculate the variance of the expected reward or even produce the entire probability distribution for each starting state. Finally, we can include the decision maker's risk aversion if the criteria of expected value is considered extreme.

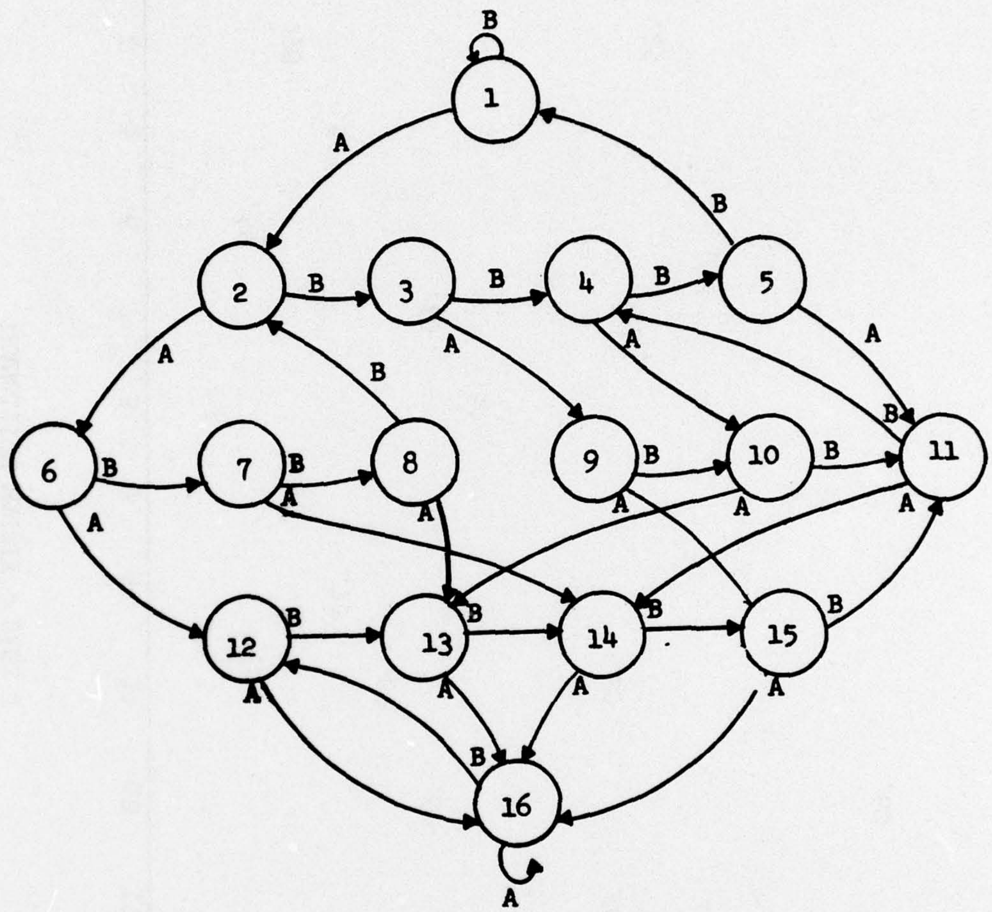
We feel the economics possible through this altered procedure merit a full scale field test.

Appendix 1

State	Speed	Altitude	Maneuver	IRCM
		Factors		
1	slow	low	no	no
2	slow	low	no	yes
3	slow	low	yes	no
4	slow	high	no	no
5	fast	low	no	no
6	slow	low	yes	yes
7	slow	high	no	yes
8	fast	low	no	yes
9	slow	high	yes	no
10	fast	low	yes	no
11	fast	high	no	no
12	slow	high	yes	yes
13	fast	low	yes	yes
14	fast	high	no	yes
15	fast	high	yes	no
16	fast	high	yes	yes

Appendix 2

TRANSITION DIAGRAM - CASE I



APPENDIX 3

TRANSITION MATRIX - CASE I

i = \ j =	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	.10	.90														
2			.35			.65										
3				.30					.70							
4					.85					.75						
5	.20										.80					
6							.40					.60				
7								.35						.65		
8		.30											.70			
9										.30				.70		
10											.25		.75			
11				.25										.75		
12													.45			.55
13														.40		.60
14															.40	.60
15															.50	.50
16												.60				.40

Appendix 4

Utility Matrix

State	Benefits Pass	Fail
1	0	50
2	5	0
3	4	0
4	3	0
5	2	0
6	15	0
7	14	0
8	13	0
9	12	0
10	11	0
11	10	0
12	20	0
13	19	0
14	18	0
15	17	0
16	25	0

(Failure at state 1, in early states is favorable)

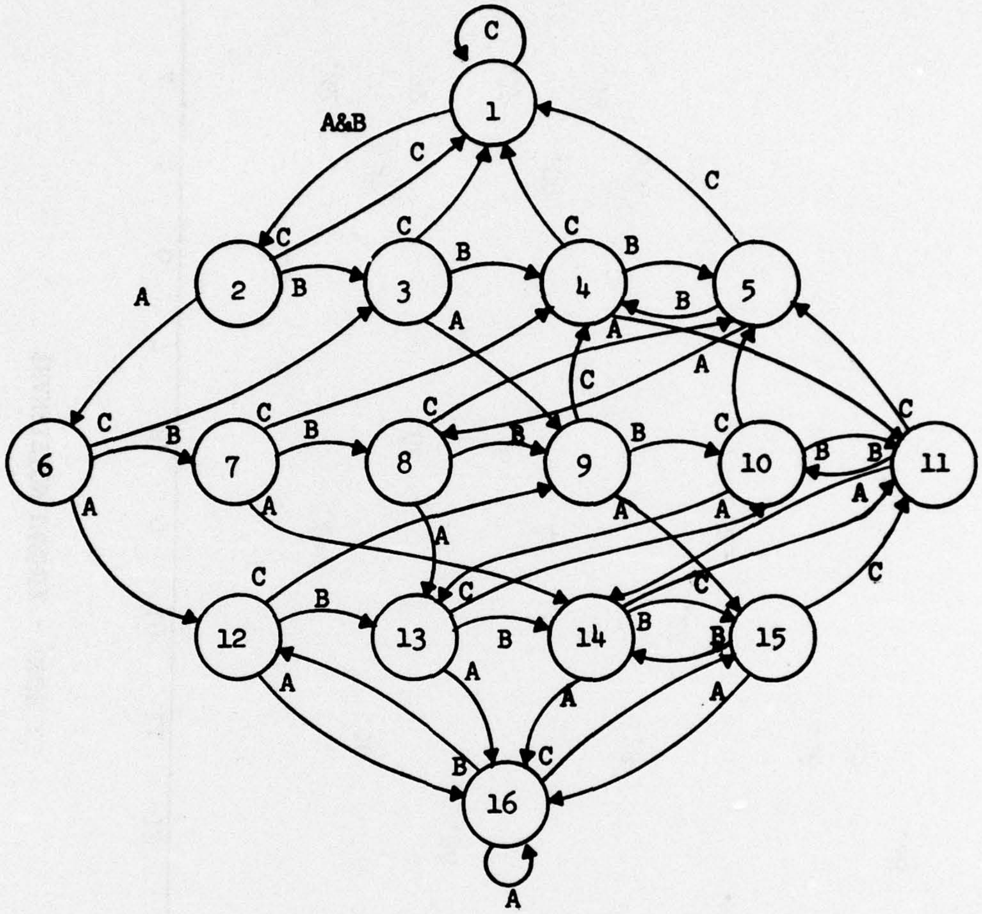
APPENDIX 5

	NUMBER OF MISSILES AVAILABLE FOR TEST PROGRAM								
	8			12			16		
	Benefit	Cost	B/C Ratio	Benefit	Cost	B/C Ratio	Benefit	Cost	B/C Ratio
1	63.0	218.1	.29	104.6	320.4	.33	146.4	422.6	.35
2	69.0	209.7	.33	110.7	312.0	.35	152.5	414.2	.37
3	67.1	212.3	.35	108.8	314.6	.346	150.6	416.8	.361
4	67.8	216.2	.314	109.5	318.4	.344	151.3	420.6	.360
5	64.5	217.5	.297	106.1	319.8	.332	147.9	422.0	.35
6	81.0	205.8	.394	122.8	308.1	.399	164.6	410.3	.401
7	79.6	207.3	.384	121.3	309.5	.392	163.1	411.7	.396
8	78.2	209.2	.373	119.9	311.4	.385	161.8	413.6	.391
9	77.7	206.6	.376	119.5	308.9	.387	161.3	411.1	.392
10	79.8	210.4	.379	121.6	312.6	.389	163.4	414.8	.394
11	75.8	210.9	.359	117.6	313.1	.376	159.4	415.3	.383
12	84.3	204.1	.413	126.1	306.4	.412	167.9	408.6	.411
13	84.0	204.9	.410	125.8	307.1	.410	167.7	409.3	.409
14	82.4	203.8	.404	124.2	306.0	.406	166.0	408.2	.407
15	79.7	201.0	.397	121.5	303.2	.401	163.3	405.5	.403
16	83.6	204.9	.408	125.4	307.1	.408	167.2	409.3	.409

(THOUSANDS)

Appendix 6

TRANSITION DIAGRAM - CASE II



APPENDIX 7

TRANSITION MATRIX - CASE II

i = \ j =	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	.05	.95														
2	.12		.46			.42										
3	.09			.42					.49							
4	.06				.38						.56					
5	.04			.32				.64								
6			.16				.48					.36				
7				.12				.46						.42		
8					.09				.42				.49			
9				.09						.42					.49	
10					.06						.38			.56		
11					.04					.28				.66		
12									.20				.50			.30
13										.16				.48		.36
14											.16				.48	.36
15											.25			.50		.25
16												.48			.36	.16

APPENDIX 8

NUMBER OF MISSILES AVAILABLE FOR TEST PROGRAM

	4 PAIRS			6 PAIRS			8 PAIRS		
	Benefit	Cost	B/C Ratio	Benefit	Cost	B/C Ratio	Benefit	Cost	B/C Ratio
1	31.8	235.8	.135	61.9	344.4	.1797	96.0	446.5	.215
2	44.0	221.7	.148	76.9	326.4	.236	111.9	426.7	.262
3	44.7	226.0	.148	78.4	327.1	.240	113.6	425.5	.267
4	46.1	224.6	.205	80.0	323.4	.247	114.9	419.8	.274
5	46.7	219.6	.212	80.1	316.9	.253	114.2	411.2	.278
6	69.6	210.3	.331	105.3	311.3	.338	141.4	410.0	.345
7	70.6	212.4	.332	106.7	312.1	.342	142.8	410.1	.348
8	71.4	213.7	.334	107.6	312.7	.344	144.0	410.6	.351
9	68.2	213.6	.319	104.4	312.2	.334	140.6	409.8	.343
10	70.5	213.9	.329	107.0	312.1	.343	143.2	409.4	.350
11	69.6	210.6	.331	105.3	306.8	.343	140.9	402.5	.350
12	78.7	197.7	.398	115.6	296.4	.390	152.4	394.7	.386
13	79.0	198.7	.397	115.7	296.9	.389	152.3	394.9	.385
14	75.8	197.2	.384	112.6	295.3	.381	149.1	393.1	.379
15	72.5	195.8	.370	109.1	293.7	.371	145.6	391.3	.372
16	74.2	185.2	.400	117.3	284.0	.392	148.0	382.2	.387

(THOUSANDS)